Professional Blackjack Analyzer

Technical Report

PBA Technical Report 1

Validity Testing of Progression Systems in Blackjack Simulators

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1. Introduction

This brief Technical Report is in response to the question of how one demonstrates that a progression-based blackjack system is being correctly simulated by a simulation code. While one can always crosscheck the output of one simulator versus another to gain confidence in the results, it is always useful to have a few tests where a result can be found by means other than a simulation. This report provides some examples and benchmarks to help determine whether a blackjack progression system is being correctly simulated.

2. The Simple Martingale

The Martingale system is perhaps the simplest example of a progression betting system. It works as follows: if the player loses a hand, then the bet is reduced, and the player wins a hand, the bet is increased. While this system does not change the edge associated with a game, it does change the variance and the average bet. The effect on the average bet in particular can be calculated directly, without the need of a simulation (see the appendix to this report), and thus can be used to check that the simulator is placing the correct bets.

In the benchmark test that follows, we refer to the simulator used in Professional Blackjack Analyzer (PBA) 4.3.7, although the analysis holds true for any simulator capable of simulating a progression based system. The screen shots and description of setting up the analysis, however, are specific to PBA.

We start with a standard game of blackjack, having the following set of rules:

```
Six deck shoe
Insurance offered
Dealer stands on soft-17
With dealer blackjack, player...
      Loses one bet max
On ties, player pushes, except...
      Loses with 22-26
Resplit to make four hands
Resplit aces not allowed
OK to split unlike 10s, like J-Q
Check hole card under 10, A
Double any hard total
Soft doubling allowed
After splitting...
      No hard doubling
```

No counting system is needed for this benchmark, but the Martingale betting system does need to be entered. This can be done in PBA by selecting the Betting Systems dialog from the Rules Editor Module, selecting a pattern-based system, and entering the system shown in Figure 1. In this system, the minimum bet is $10, the maximum bet is $160, the player doubles a bet after a win (W) and halves a bet after a loss (L). *Note that in PBA 4.3.7, ties are treated as losses in the Simulator Module!*

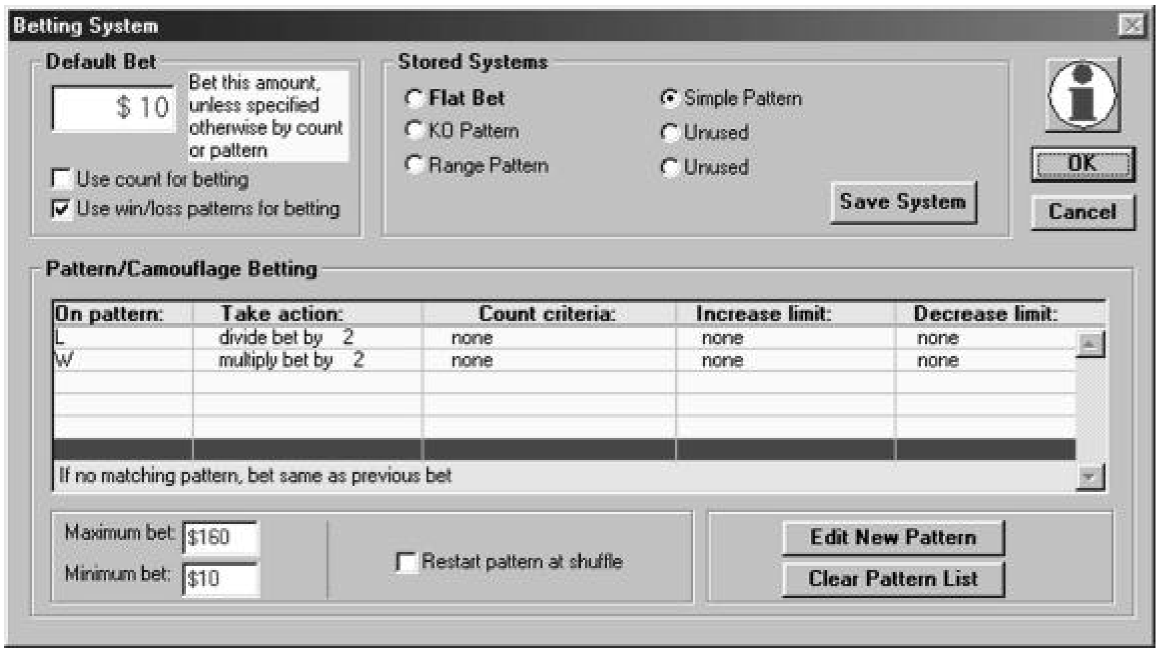

Figure 1: A Martingale system as entered in PBA.

Note that with this system there are only five possible bets: $10, $20, $40, $80, and $160. Thus, if this system is followed over many hands of play, the average bet can be determined using Equation (9) from the Appendix, once the probabilities of winning versus losing have been determined.

The game and betting system described above were simulated in PBA 4.3.7, yielding the this report:

Simulation Results for Simple Progression System:

```
S>-----------------------------------------------------
Edge:    -0.56
Standard error:    0.04
Sigma:   1.64
Desirability index (DI):   0.00
Number of hands simulated:   20,010,000
Average initial bet: $43.95
Profit per hand:    -$0.25
Standard deviation per hand:    $72.24
Profit per 100 hands:    -$24.50
Standard deviation per 100 hands:    $722.36
Win percentage of total hands:    43.28%
Loss percentage of total hands:    47.90%
Tie percentage of total hands:    8.82%
S>-----------------------------------------------------
```

Note that the percentages of wins, losses, and ties are summarized in the report, so that we can determine the ratio of wins to losses. Since W=.4328 and L=.5672 (remember ties are treated as losses here), x=W/L=0.7630. Equation (9) predicts that the average bet should be $44.21 based on the initial $10 bet with five progressions. As shown in the report above, the simulator in PBA finds the average bet to be $43.63, well within the range of error allowed by the 870,000 hands simulated. This simple test verifies that the simulator places the correct bets based on patterns.

3. General Remarks on Progression Systems

In the example shown in Section 2, the edge associated with the game was negative; that is, the progression system did not lead to a profitable game for the player. In fact, comparing the progression betting system to a flat betting system (results below) shows that the two yield the same edge, as shown below:

Simulation Results for Flat Bet System:

```
S>-----------------------------------------------------
Edge:    -0.56
Standard error:    0.03
Sigma:   1.13
Desirability index (DI):   0.00
Number of hands simulated:   20,000,000
Average hands per shoe:   51.90
Average initial bet: $10.00
Profit per hand:    -$0.06
Standard deviation per hand:    $11.31
Profit per 100 hands:    -$5.56
Standard deviation per 100 hands:    $113.11

Win percentage of total hands:    43.28%
Loss percentage of total hands:    47.89%
Tie percentage of total hands:    8.82%
S>-----------------------------------------------------
```

The simulations indicate that following this progression scheme does not alter the rate of winning. So what changed? Comparing the two tables, it can be seen that, on average, the initial bet in the progression system is about four times as large, and the standard deviation per hand jumps from $11.31 for flat betting to $72.24 for the progression system.

It should be pointed out that the systems considered here are both losing systems in the long run, as the edge is –0.56. Multiplying the edge by the average bet gives the net profit for the player, which, being negative in both cases, means the player loses money with either system. For a progression system to be a winning system, the edge would need to be greater than zero. Combining progression systems with counting systems can lead to some interesting possibilities for winning systems with built-in camouflage, and these may be considered in a later report.

Appendix: Derivation of the Average Bet for Martingale Systems

The best-known progression system perhaps is the "Martingale," in which a bet is increased after a win, and decreased after a loss. It is rather easy to obtain what the average bet associated with such a system should be, by considering the probabilities associated with arriving at given bets.

Suppose the system under consideration allows a limited set of possible bets $\{b_i\}$ that may be countably infinite, where $b_i > b_j$ if i>j. After N plays, there will be a probability P_i^N that the player has placed the bet b_i. The average bet $\bar{b}$ after N plays is thus

$$\bar{b}^N = \sum_i P_i^N b_i \,. \qquad (1)$$

In this game, there is a probability W that the player wins, and thus raises the bet, a probability L that the player loses and thus reduces the bet, and a probability T that the player ties and does not change the bet. The probabilities after N+1 hands can thus be related to the probabilities after N hands by the following relation:

$$P_i^{N+1} = W \cdot P_{i-1}^N + L \cdot P_{i+1}^N + T \cdot P_i^N \qquad (2)$$

where W+L+T=1 and the sum of all P_i^N for a given N also equals one.

After a large number of plays, it is expected that the probabilities will not change much from one hand to the next; that is, the P_i^N converge for N large. The N superscript is thus no longer needed in Equation (2), and a recursion relation can be written:

$$P_{i+1} = \frac{1-T}{L} \cdot P_i + \frac{W}{L} \cdot P_{i-1} \qquad (3)$$

Equation (3) is a second-order difference equation, which can be solved with two additional conditions. The first condition is

$$\sum_i P_i = 1 \,. \qquad (4)$$

A second condition can be found by insisting that there is a minimum bet b_1. If the player loses a bet of size b_1, then the bet is not reduced, but instead b_1 is placed again. Of course, if the player wins with the bet b_1 placed, then the following bet is b_2. With the two conditions, Equation (3) can be solved, with the solution:

$$P_i = C\left(\frac{W}{L}\right)^i \tag{5}$$

where C is a normalization constant. If the largest bet is not restricted, the series can be summed, and the explicit solution can be written:

$$P_i = \left(\frac{W}{L}\right)^{i-1}\left(1-\frac{W}{L}\right) \tag{6}$$

Note that for Equations (5,6) to make sense, W must be smaller than L. If W=L, then the "average" bet does not converge to a fixed value, but increases with increasing N.

If there is an upper bound to the bets, such that b_M is the largest bet that can be placed, then the solution for the probabilities becomes

$$P_i = x^{i-1}\left(\frac{1-x}{1-x^M}\right) \tag{7}$$

where x=W/L. Having derived the probabilities, the average bet can be determined from Equation (1) as

$$\bar{b} = \left(\frac{1-x}{1-x^M}\right)\sum_i b_i \cdot x^{i-1} \ . \tag{8}$$

Consider the simple system where if the player wins, the bet is doubled, and if the player loses, the bet is halved. This is the most common Martingale progression, and the series in Equation (8) can be summed to obtain the final expression

$$\bar{b} = b_1\left(\frac{1-x}{1-x^M}\right)\left(\frac{(2x)^M - 1}{2x-1}\right) \tag{9}$$